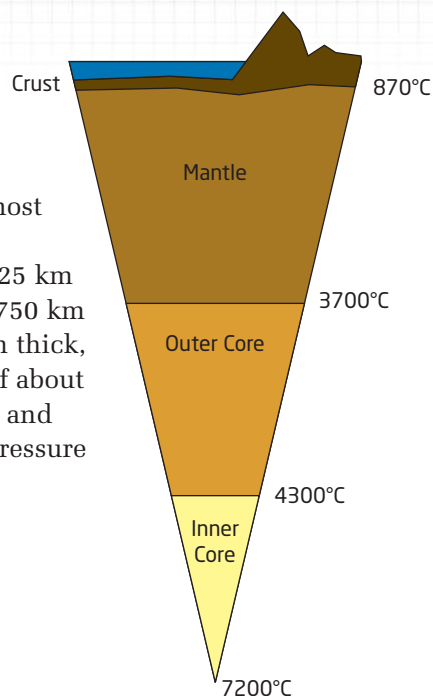


5.2

Partial Variation

Earth is made up of several distinct layers. Beneath the oceans, the outermost layer, or crust, is 5 km to 12 km thick. Below the continents, Earth's crust is 25 km to 90 km thick. The mantle is about 2750 km thick. The outer core is about 2260 km thick, while the inner core has a thickness of about 1228 km. The deeper layers are hotter and denser because the temperature and pressure inside Earth increase with depth.



Investigate

What is the relationship between temperature and depth?

The temperature of Earth's crust increases by about 145°C for every kilometre below the oceans. The temperature increases by about 21.75°C for every kilometre below the continents.

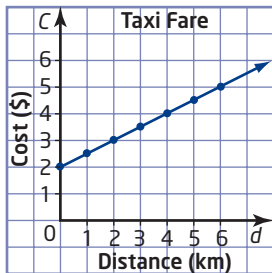
- Starting at a temperature of 10°C at the surface of Earth's crust, make a table showing the depth and temperature of Earth's crust under the ocean and under the continents, at depths between 0 km and 5 km.

Depth (km)	Temperature Under Oceans ($^{\circ}\text{C}$)	Temperature Under Continents ($^{\circ}\text{C}$)

- Plot temperature versus depth for your data under the oceans.
 - On the same grid, plot temperature versus depth for your data under the continents.
- Compare and contrast the two graphs.
- How do these graphs differ from those you made for direct variation in Section 5.1?

5. Consider the equation $T = 145d + 10$, where T represents the temperature, in degrees Celsius, under the oceans and d represents the depth, in kilometres.
 - a) Substitute $d = 1$ and calculate T . Repeat for $d = 2, 3, 4, 5$. Compare the results with those you obtained in step 1.
 - b) Explain why this equation works.
 - c) Write a similar equation relating the temperature under the continents with depth.
6. **Reflect** Describe the parts of each equation and how they relate to the data in your table and graph.

The graph illustrates the total cost, C , in dollars, of a taxi fare for a distance, d , in kilometres.



The fixed cost of \$2 represents the initial meter fare. The distance travelled by taxi changes, or is *variable*, depending on the passenger's destination. For this reason, the variable cost is \$0.50 times the distance.

$$C = 2 + 0.5d$$

fixed cost variable cost

The graph is a straight line, but it does not show a direct variation because the line does not pass through the origin. This type of relationship is called a **partial variation**. Another way to describe this partial variation is to say that " C varies partially with d ."

In general, if y varies partially with x , the equation is of the form $y = mx + b$, where m and b are constants, and

- m represents the constant of variation
- b represents the fixed, or initial, value of y

partial variation

- a relationship between two variables in which the dependent variable is the sum of a constant number and a constant multiple of the independent variable

Example 1 Graph a Partial Variation

- a) Copy and complete the table of values given that y varies partially with x .
- b) Identify the initial value of y and the constant of variation from the completed table. Write an equation relating y and x in the form $y = mx + b$.
- c) Graph this relation.
- d) Describe the graph.

x	y
0	6
1	9
2	
3	15
4	
	27

Solution

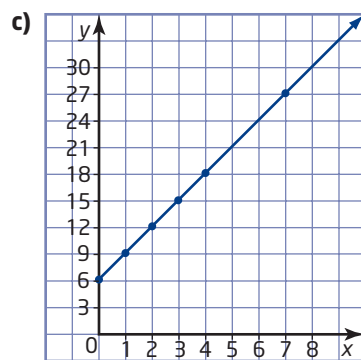
- a) As x changes from 0 to 1, y changes from 6 to 9. Therefore, y increases by 3 as x increases by 1.

x	y
0	6
1	9
2	12
3	15
4	18
7	27

The pattern of increasing the y -values by 3 checks for the other values that were given.

- b) The initial value of y occurs when $x = 0$. The initial value of y is 6. As x increases by 1, y increases by 3. So, the constant of variation is 3.

Use $b = 6$ and $m = 3$ to obtain the equation $y = 3x + 6$.



- d) The graph is a straight line that intersects the y -axis at the point $(0, 6)$. The y -values increase by 3 as the x -values increase by 1.

Example 2 School Awards Banquet

A school is planning an awards banquet. The cost of renting the banquet facility and hiring serving staff is \$675. There is an additional cost of \$12 per person for the meal.

- Identify the fixed cost and the variable cost of this partial variation.
- Write an equation to represent this relationship.
- Use your equation to determine the total cost if 500 people attend the banquet.



Solution

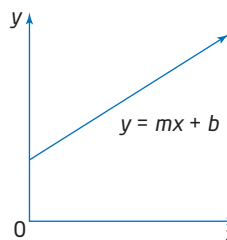
- The fixed cost is \$675. The variable cost is \$12 times the number of people.
- Let C represent the total cost, in dollars. Let n represent the number of people attending.
Multiply the number of people by 12 and add 675.
$$C = 12n + 675$$
- Substitute $n = 500$.

$$\begin{aligned} C &= 12(500) + 675 \\ &= 6675 \end{aligned}$$

The total cost for 500 people is \$6675.

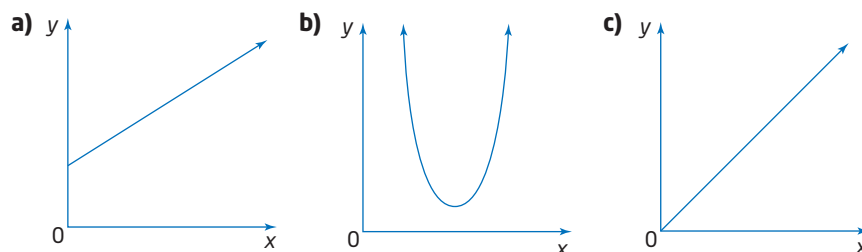
Key Concepts

- A partial variation has an equation of the form $y = mx + b$, where b represents the fixed, or initial, value of y and m represents the constant of variation.
- The graph of a partial variation is a straight line that does not pass through the origin.



Communicate Your Understanding

- C1** Classify each graph as a direct variation, a partial variation, or neither. Justify your answer.



- C2** The cost to repair a television set is made up of a service charge of \$50, which covers the travel time and gas for the repairperson, plus \$40/h. Describe the steps involved in developing a partial variation equation that relates the cost and the time required to complete the repairs.

- C3** The table models a partial variation. Discuss how you can use the table to find m and b in the equation $y = mx + b$.

x	y
0	10
1	14
2	18
3	22
4	26

Practise

1. Identify each relation as a direct variation, a partial variation, or neither. Justify your answer.

a) $y = 3x$ b) $y = 2x + 1$
c) $C = 20n + 500$ d) $d = 5t$

For help with questions 2 and 3, see Example 1.

2. a) Copy and complete the table of values given that y varies partially with x .
b) Identify the initial value of y and the constant of variation from the table.
c) Write an equation relating y and x in the form $y = mx + b$.
d) Graph the relation.
e) Describe the graph.

x	y
0	5
1	10
2	
3	20
4	
	40

3. a) Copy and complete the table of values given that y varies partially with x .
- b) Identify the initial value of y and the constant of variation from the table.
- c) Write an equation relating y and x in the form $y = mx + b$.
- d) Graph the relation.
- e) Describe the graph.

x	y
0	-2
1	3
2	
3	13
4	
	33

For help with questions 4 and 5, see Example 2.

4. A small pizza costs \$7.00 plus \$1.50 per topping.
- a) Identify the fixed cost and the variable cost of this partial variation.
- b) Determine the equation relating the cost, C , in dollars, and the number of toppings, n .
- c) Use the equation to determine the cost of a small pizza with five toppings.
5. A class is planning a field trip to an art gallery. The cost of renting a bus is \$250. There is an additional cost of \$4 per student for the entrance fee.
- a) Identify the fixed cost and the variable cost of this partial variation.
- b) Write an equation relating the cost, C , in dollars, and the number of students, n .
- c) Use your equation to determine the total cost if 25 students attend.

Connect and Apply

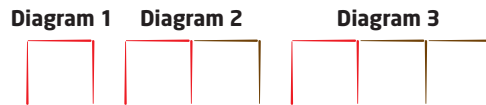
6. A fitness club offers two types of monthly memberships:
- membership A: \$4 per visit
 - membership B: a flat fee of \$12 plus \$2 per visit
- a) Graph both relations for 0 to 10 visits.
- b) Classify each relation as a direct variation or a partial variation.
- c) Write an equation relating the cost and the number of visits.
- d) Compare the monthly membership costs. When is membership A cheaper than membership B? When is membership B cheaper than membership A?



7. The table shows the amount a printing company charges for advertising flyers.

Number of Flyers, n	Cost, C (\$)
0	100
100	120
200	140
300	160

- Identify the fixed cost this company charges for producing the flyers. What do you think this amount might represent?
 - Determine the variable cost for producing one flyer. Explain how you found this.
 - Write an equation representing the price for the flyers.
 - What is the cost to produce 1000 flyers?
 - How many flyers can be produced for \$280?
8. **Chapter Problem** Toothpick patterns are popular puzzles. Here is an example of a toothpick pattern.

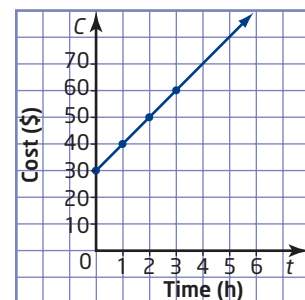


- Write an equation relating the diagram number and the number of toothpicks. Is this a partial variation? Explain.
- Use your equation to determine the number of toothpicks in Diagram 20.

Did You Know?

Scuba divers will ascend slowly to the surface to avoid decompression sickness, or the bends. After deep or long dives, a scuba diver needs to undergo decompression by returning to the surface slowly and in stages.

- At the surface of a lake, a scuba diver experiences 102.4 kPa (kilopascal) of pressure. As the diver descends, the pressure increases by 101.3 kPa for every 10 m.
 - Write an equation that relates the pressure experienced by a diver and the depth that the diver has descended.
 - Divers must be aware of nitrogen narcosis, which occurs when too much nitrogen dissolves in the blood. Narcosis becomes possible when the diver is exposed to a pressure of about 400 kPa. At what depth does the danger from narcosis begin?
- Describe a situation that might lead to this graph.



11. At 12:05 P.M., a parachutist was 8000 m above the ground.
 At 12:06 P.M., the parachutist was 7750 m above the ground.
 At 12:07 P.M., the parachutist was 7500 m above the ground.
- Graph this relation.
 - Find the average rate of descent, in metres per minute.
 - Write an equation for this relation.

Achievement Check

12. A theatre company produced the musical *Cats*. The company had to pay a royalty fee of \$1250 plus \$325 per performance. The same theatre company also presented the musical production of *Fame* in the same year. For the production of *Fame*, they had to pay a royalty fee of \$1400 plus \$250 per performance.
- Write an equation that relates the total royalties and the number of performances for each musical.
 - Graph the two relations on the same grid.
 - When does the company pay the same royalty fee for the two productions?
 - Why do you think the creators of these musicals would set royalties in the form of a partial variation instead of a direct variation?

Extend

13. In Earth's atmosphere, the speed of sound can be approximated using partial variation. The speed of sound is approximately 331 m/s at 0°C and approximately 343 m/s at 20°C.
- What is the approximate speed of sound at
 - 30°C?
 - 30°C?
 - Jenny yells out "Hello" in a canyon when the air temperature is -10°C. It takes 1.4 s to hear her echo. How far away is the wall of the canyon?
14. A battery was recharged, remained fully charged, and then slowly lost its charge, as shown in the table.

Time (h)	0	5	10	15	20	25	30	35	40	45	50	55	60
Charge (%)	92	94	96	98	100	100	100	100	95	90	85	80	75

- Graph the battery's charge over time.
- Determine an appropriate set of equations for the charge of the battery.
- What was the remaining charge after
 - 12 h?
 - 26 h?
 - 71 h?

